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Notes and Practice (Piece-wise and Infinity)

Name _____ Date _____

requested limits. If you use the graphing calculator, write "calc" and draw a sketch

$$\frac{x^2 - 2x}{x + 1} = \frac{9 - 4}{4} = \boxed{\frac{3}{4}}$$

2) $\lim_{x \rightarrow 1^+} \frac{(x^2 + 1)(x^2 - 1)}{x - 1} = \frac{(x^2 + 1)(x + 1)}{2 \cdot 2}$

~~$(x^2 + 1)(x + 1)$~~

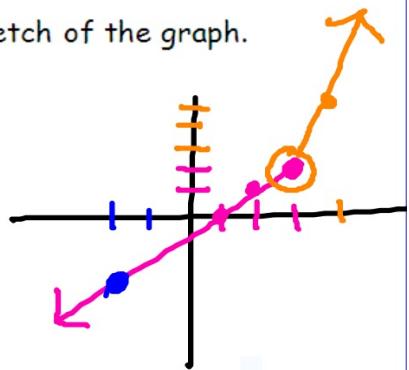
$\boxed{4}$

$$\frac{3 - x}{x^2 - 2x - 8} = \boxed{-\infty}$$

4) $\lim_{x \rightarrow 5} 3 = \boxed{3}$

Find the requested limits for the piecewise functions. Draw a sketch of the graph.

5) $f(x) = \begin{cases} x-1 & \text{if } x \leq 3 \\ 3x-7 & \text{if } x > 3 \end{cases}$

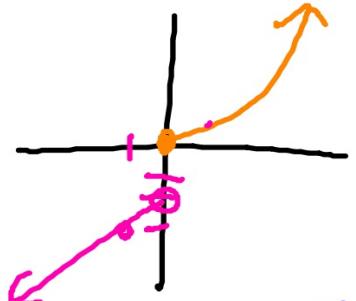


$$\lim_{x \rightarrow 3^-} f(x) = 2$$

$$\lim_{x \rightarrow 3^+} f(x) = 2$$

$$\lim_{x \rightarrow 3} f(x) = 2$$

6) $g(t) = \begin{cases} t^2 & \text{if } t \geq 0 \\ t-2 & \text{if } t < 0 \end{cases}$



$$\lim_{t \rightarrow 0^-} g(t) = -2$$

$$\lim_{t \rightarrow 0^+} g(t) = 0$$

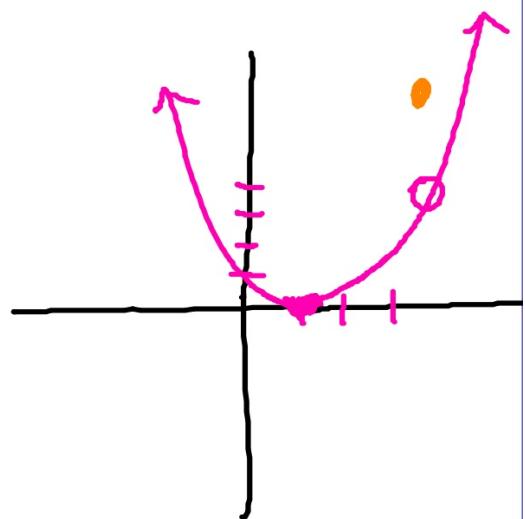
$$\lim_{t \rightarrow 0} g(t) = \text{D.N.E}$$

$$7) f(x) = \begin{cases} x^2 - 2x + 1 & \text{if } x \neq 3 \\ 7 & \text{if } x = 3 \end{cases}$$

$9 - (6 + 1)$
 4

$s(x-1)(x-1)$

$$\lim_{x \rightarrow 3} f(x) = \boxed{4}$$



$$8) f(x) = \begin{cases} -x^2 + 3 & \text{if } x \leq -2 \\ -2x + 2 & \text{if } -2 < x < 5 \\ (x-4)^2 + 4 & \text{if } x \geq 5 \end{cases}$$

$$f(2) = -1$$

$$\lim_{x \rightarrow -2^-} f(x) = -1$$

$$\lim_{x \rightarrow 0^-} f(x) = 2$$

$$\lim_{x \rightarrow 5^-} f(x) = -8$$

$$f(0) = 2$$

$$\lim_{x \rightarrow -2^+} f(x) = 6$$

$$\lim_{x \rightarrow 0^+} f(x) = 2$$

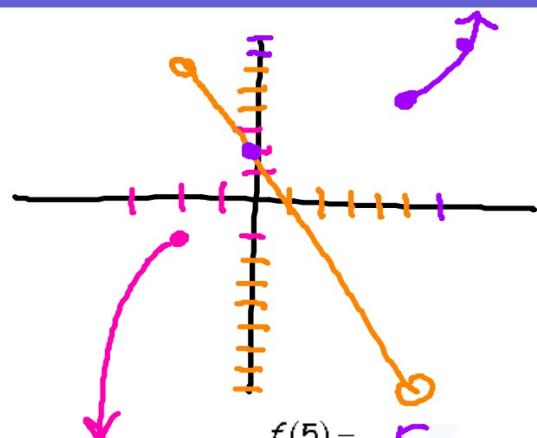
$$\lim_{x \rightarrow 5^+} f(x) = 5$$

$$f(5) = 5$$

$$\lim_{x \rightarrow 2} f(x) = \text{DNE}$$

$$\lim_{x \rightarrow 0} f(x) = 2$$

$$\lim_{x \rightarrow 5} f(x) = \text{DNE}$$



Infinity Limits

9. If the highest power in the numerator < the highest power in the denominator, the limit = 0

$$\lim_{x \rightarrow \infty} \frac{x^2 + 3x - 2}{x^4 - 6x + 5} = 0$$

10. If the highest power in the numerator > the highest power in the denominator, the limit = ∞

$$\lim_{x \rightarrow \infty} \frac{x^3 + 2}{x^2 - 8} = \infty$$

11. If the highest power in the numerator = the highest power in the denominator, the limit = $\frac{2}{5}$

$$\lim_{x \rightarrow \infty} \frac{2x^2 + 7x + 1}{5x^2 + 7} = \frac{2}{5}$$

ratio of
coefficients
of highest
exponent

$$\lim_{x \rightarrow \infty} 2^{-x} = \frac{1}{2^{\infty}} = \frac{1}{\infty}$$

$$\lim_{x \rightarrow \infty} \frac{3x^2 - 5x + 2}{x^2 - 9}$$

3

$$13. \lim_{x \rightarrow 3^-} \frac{5}{x-3} = -\infty$$

$$14. \lim_{x \rightarrow \infty} \frac{x+4}{\sqrt{2x^2 + 7}}$$

$$\frac{x}{\sqrt{2x^2}} = \frac{x}{\sqrt{2} \cdot \sqrt{x^2}} = \frac{1}{\sqrt{2}}$$

$$16. \lim_{x \rightarrow -1} \frac{(x+1)(x-5)}{(x-1)(x+1)} = \frac{-6}{-2} = 3$$

$$17. \lim_{x \rightarrow 2} 3x^2 - 4$$

8

$$18. \lim_{x \rightarrow \infty} \frac{\sqrt{2x^2 + 5x}}{3x - 4}$$

$$\frac{\sqrt{2}}{3}$$

$$19. \lim_{x \rightarrow -5} 4$$

$$4$$

$$20. \lim_{x \rightarrow \infty} \frac{4}{\sqrt{x}}$$

$$\begin{array}{c} 0 \\ | \\ f \\ | \\ \text{TI} \end{array}$$

$$21. \lim_{x \rightarrow 6} \frac{x^2 - 36}{x - 6}$$

$$|2$$

$$22. \lim_{x \rightarrow 1} \frac{x^3 - 1}{x - 1}$$

$$3$$

$$23. \lim_{x \rightarrow \frac{\pi}{2}} \sin x$$

$$1$$

$$24. \lim_{x \rightarrow 2} \frac{x}{x^2 - 4} =$$

DNE

$$25. \lim_{x \rightarrow 0^-} \frac{x}{|x|} =$$

$$-1$$

$$26. \lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x^2 - 4}$$

$$(x-2)(x+2)$$

Limits of piecewise functions and one-sided limits: Find the requested limits.

27. $f(x) = \begin{cases} x+2 & \text{if } x \leq 3 \\ x+4 & \text{if } x > 3 \end{cases}$

$$f(3) = 5$$

$$\lim_{x \rightarrow 10} f(x) = 14$$

$$\lim_{x \rightarrow 2} f(x) = 4$$

$$\lim_{x \rightarrow 3^-} f(x) = 5$$

$$\lim_{x \rightarrow 3^+} f(x) = 7$$

$$\lim_{x \rightarrow 3} f(x) = \text{DNE}$$

$$28. \ g(t) = \begin{cases} t^2 & \text{if } t > 1 \\ 2 & \text{if } t \leq 1 \end{cases}$$

$$g(5) = 25$$

$$g(0) = 2$$

$$g(1) = 2$$

$$\lim_{x \rightarrow -4} g(t) = 2$$

$$\lim_{x \rightarrow 7} g(t) = 49$$

$$\lim_{x \rightarrow 2^+} g(t) = 4$$

$$\lim_{x \rightarrow 1^-} g(t) = 2$$

$$\lim_{x \rightarrow 1^+} g(t) = 1$$

$$\lim_{x \rightarrow 1} g(t) = \text{DNE}$$

$$29. f(x) = \begin{cases} 2 & \text{if } x \leq -1 \\ -x + 1 & \text{if } -1 < x < 1 \\ \frac{1}{2}x - \frac{1}{2} & \text{if } x \geq 1 \end{cases}$$

$$f(5) = 2$$

$$\lim_{x \rightarrow 0} f(x) = 1$$

$$\lim_{x \rightarrow 1^-} f(x) = 0$$

$$\lim_{x \rightarrow 1^-} f(x) = 2$$

$$f(4) = 1.5$$

$$\lim_{x \rightarrow 3} f(x) = 2$$

$$\lim_{x \rightarrow 1^+} f(x) = 0$$

$$\lim_{x \rightarrow 1^+} f(x) = 2$$

$$f(-1) = 2$$

$$\lim_{x \rightarrow 4} f(x) = 1.5$$

$$\lim_{x \rightarrow 1} f(x) = 0$$

$$\lim_{x \rightarrow -1} f(x) = 2$$

$$30. f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ x & \text{if } 0 < x < 1 \\ 1 - x^2 & \text{if } x \geq 1 \end{cases}$$

$$f\left(\frac{3}{4}\right) = \underline{\underline{3/4}}$$

$$f(4) = \underline{\underline{-15}}$$

$$f(0) = \underline{\underline{0}}$$

$$\lim_{x \rightarrow 0} f(x) = \underline{\underline{0}}$$

$$\lim_{x \rightarrow -3} f(x) = \underline{\underline{9}}$$

$$\lim_{x \rightarrow 4} f(x) = \underline{\underline{-15}}$$

$$\lim_{x \rightarrow 1^-} f(x) = \underline{\underline{1}}$$

$$\lim_{x \rightarrow 1^+} f(x) = \underline{\underline{0}}$$

$$\lim_{x \rightarrow 1} f(x) = \underline{\underline{DNE}}$$

$$\lim_{x \rightarrow 2} f(x) = \underline{\underline{-3}}$$

$$\lim_{x \rightarrow -4} f(x) = \underline{\underline{16}}$$

$$\lim_{x \rightarrow 6} f(x) = \underline{\underline{-35}}$$